UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498DJ Principles of Signal Analysis

Problem Set 3

Fall 2011

Assigned: 9/23/2011 **Due:** 9/30/2011

Problem 3.1

Let's start with the following signal:

$$x_1[n] = \begin{cases} 1 & 0 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

(a) The discrete-time Fourier transform of $x[n]$ can be written as

$$X_1(\omega) = e^{-j\omega L} A(\omega),$$

for some real-valued L and real-valued $A(\omega)$. Find L and $A(\omega)$.

(b) Find the form of the 8-point DFT, $X_1[k]$, for $0 \leq k \leq 7$, by just sampling your answer to part (a) at the right values of ω.

(c) Evaluate your answer to part (b) in order to find the actual numerical values of $X_1[k]$, for $0 \leq k \leq 7$.

(d) Show that your answer to part (c) satisfies the conjugate-symmetry property.

Problem 3.2

Consider the following two signals:

$$x_2[n] = \begin{cases} 1 & 0 \leq n \leq 6 \\ 0 & \text{otherwise} \end{cases}$$

$$y_2[n] = \begin{cases} 1 & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the DTFT of either of these two signals (hint: you can just apply your answer from problem 1, but with different constants). Then use the time-shift property to find the DTFT of the other signal.

Problem 3.3

Consider the following two signals:

$$x_3[n] = \begin{cases} 1 & -7 \leq n \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

$$y_3[n] = \begin{cases} \cos\left(\frac{5\pi n}{7}\right) & -3 \leq n \leq 3 \\ 0 & \text{otherwise} \end{cases}$$

Find the DTFT of either of these two signals (hint: you can just apply your answer from problem 2, but with different constants). Then use the modulation property to find the DTFT of the other signal (hint: use Euler's relations to express the cosine in terms of complex exponentials, then apply the modulation property). Sketch $Y_3(\omega)$ for $-\pi \leq \omega \leq \pi$.

Problem 3.4

Show that the inverse DTFT of $X_4(\omega) = 2\pi\delta(\omega - \pi)$ is $x_4[n] = (-1)^n$, i.e., the sequence

$$x_4[n] = \{1, -1, 1, -1, 1, \ldots\}$$