UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498DJ Principles of Signal Analysis

Problem Set 5

Fall 2011

Assigned: 10/24/2011 **Due:** 11/7/2011

Problem 5.1

Your friend Leo is designing a flying dragon. In order to make it fly more smoothly, he proposes to smooth all of the motor commands by passing them through the following difference equation:

$$y[n] = x[n] + 0.5x[n-1] + 0.25x[n-2] + 0.5y[n-1] - 0.25y[n-2]$$

where $x[n]$ is the original motor command, and $y[n]$ is the smoothed motor command.

(a) Does this difference equation implement an IIR or FIR filter?

(b) Compute samples of the impulse response $h[n]$ for $n = -1, 0, 1, 2, 3$ by hand, by applying the input $x[n] = \delta[n]$.

(c) Find the transfer function, $H(z) = Y(z)/X(z)$.

(d) Find and plot the poles and zeros.

(e) Is this a BIBO stable filter? [Hint: check the poles]

Problem 5.2

For the filter from problem 1, solve the following:

(a) Find the frequency response, $H_d(\omega)$.

(b) Sketch the magnitude of the frequency response.

(c) It is a lowpass, highpass, bandpass, or bandstop filter?

(d) It is a linear-phase filter?

(e) Suppose that the input is $x[n] = x_c(nT)$, and the output is converted back into continuous time according to

$$y_c(t) = \sum_{n=-\infty}^{\infty} y[n]\text{sinc}(t - nT)$$

where $1/T = 10,000$ samples/second. Find the frequency response $H(\Omega) = Y(\Omega)/X(\Omega)$ of the equivalent continuous-time system.

(f) Sketch the magnitude of the equivalent continuous-time frequency response, $|H(\Omega)|$.

Problem 5.3

Consider the FIR filter $h[n] = 0.25\delta[n] + 0.5\delta[n-1] + \sqrt{3}\delta[n-2] + 0.5\delta[n-3] + 0.25\delta[n-4]$.

(a) Calculate the frequency response, $H_d(\omega)$, of this filter.

(b) Is this a generalized linear phase filter?

(c) Is it a lowpass, highpass, bandpass, or bandstop filter?

(d) Calculate and sketch the magnitude and phase of this filter.

Problem 5.4

Tell whether the following filters are symmetric, antisymmetric, or neither, and whether or not they are generalized linear phase filters.

(a) $h[n] = [-1, 3, 3, -1]$

(b) $h[n] = [-1, 3, 3, 1]$

(c) $h[n] = [-1, -3, 3, 1]$

(d) $h[n] = [-1, 3, -3, 1]$

Problem 5.5

Design a length-8 generalized linear phase FIR lowpass filter with a cutoff frequency of $\omega_c = \pi/4$ using the window design method with a Hamming window. Give the filter coefficients as your answer. Either show your work (hand calculations) or your matlab code that computed the filter coefficients. Plot the magnitude and phase from $-\pi \leq \omega < \pi$ using Matlab (Hint: `fftshift(abs(fft(h,1000)))` will give you 1000 samples of the magnitude over $-\pi \leq \omega < \pi$)