ECE 498MH
12/6/2013

TOPICS

(1) PHASORS

PROBLEM $A\cos(\Omega t+\theta) + B\sin(\Omega t+\phi) = C\cos(\Omega t$

WHAT ARE C AND ψ?

SOLUTION $A\cos(\Omega t+\theta) = Re\left(Ae^{j\theta} e^{j\Omega t}\right)$

$$B\sin(\Omega t+\phi) = Re\left(-jBe^{j\phi} e^{j\Omega t}\right)$$

$$C\cos(\Omega t+\psi) = Re\left(\left(Ae^{j\theta} - jBe^{j\phi}\right) e^{j\Omega t}\right)$$

$$\Rightarrow \quad C = \left|Ae^{j\theta} - jBe^{j\phi}\right|$$

$$\psi = \measuredangle\left(Ae^{j\theta} - jBe^{j\phi}\right)$$

(2) FOURIER SERIES

THEOREM

IF $x(t)$ PERIODIC W/T_0, THEN $x(t) = \sum_{k=-\infty}^{\infty} X_k e^{j2\pi kt}$

WITH POWER SPECTRUM $|X_k|^2$

PROBLEM WHAT IS X_k?

SOLUTION

$\boxed{\text{CTFS}}$ $$X_k = \frac{1}{T_0} \oint x(t)\, e^{-j2\pi kt/T_0}\, dt$$

$\boxed{\text{DTFS}}$ $$X_k = \frac{1}{N_0} \sum x[n]\, e^{-j2\pi kn/N_0}$$

3 LTI SYSTEMS

THEOREM $x[n] \longrightarrow \boxed{\text{SYSTEM}} \longrightarrow y[n]$

IF SYSTEM IS LTI, $y[n] = \sum_{m=-\infty}^{\infty} h[m]x[n-m]$

PROBLEM HOW DO WE KNOW IF IT'S LTI?

SOLUTION

LINEAR

$$x_1[n] \longrightarrow y_1[n]$$

$$x_2[n] \longrightarrow y_2[n]$$

$$x_3[n] = ax_1[n] + bx_2[n] \longrightarrow y_3[n]$$

$\boxed{\text{LINEAR IFF ALWAYS}}$

$$y_3[n] = ay_1[n] + by_2[n]$$

TI

$$x_1[n] \longrightarrow y_1[n]$$

$$x_2[n] = x_1[n-d] \longrightarrow y_2[n]$$

$\boxed{\text{TI IFF ALWAYS}}$

$$y_2[n] = y_1[n-d]$$

4 IMPULSE RESPONSE, FREQ RESPONSE

THEOREM $x[n] \longrightarrow \boxed{\text{LTI}} \longrightarrow y[n] = h[n] * x[n]$

$$e^{jwn} \longrightarrow \boxed{\text{LTI}} \longrightarrow H(w)\, e^{jwn}$$

$$A \cos(wn+\theta) \longrightarrow \boxed{\text{LTI}} \longrightarrow A \cdot |H(w)| \cos(wn+\theta+\angle H(w))$$

PROBLEM WHAT IS $h[n]$? WHAT IS $H(w)$?

SOLUTION

EITHER

$x[n] = \delta[n] \longrightarrow \boxed{LTI} \longrightarrow y[n] = h[n]$

OR

$x[n] = e^{jwn} \longrightarrow \boxed{LTI} \longrightarrow y[n]$

$\boxed{THEN}$

$$H(w) = \sum_{m=-\infty}^{\infty} h[m] e^{-jwm}$$

$$h[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} H(w) e^{jwn} dw$$

[5] Z TRANSFORM

PROBLEM : HOW CAN YOU DO $e^{jwn} \longrightarrow \boxed{LTI}$

FOR $y[n] + a_1 y[n-1] + \cdots + a_N y[n-N] = b_0 x[n] + \cdots + b_M$

SOLUTION

$$\mathcal{Z}\{af[n] + bg[n]\} = aF(z) + bG(z)$$

$$\mathcal{Z}\{y[n-1]\} = z^{-1} Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + \cdots + b_M z^{-M}}{1 - a_1 z^{-1} - \cdots - a_N z^{-N}}$$

$$H(w) = \frac{b_0 + b_1 e^{-jw} + \cdots + b_M e^{-jwM}}{1 - a_1 e^{-jw} - \cdots - a_N e^{-jwN}}$$

[6] WINDOWING

PROBLEM

$$y[n] = x[n] * h[n]$$

$$z[n] = x[n]\, w[n]$$

SOLUTION

$$Y(w) = X(w) H(w)$$

$$Z(w) = \frac{1}{2\pi} X(w) * W(w)$$

7 UPSAMPLING & DOWNSAMPLING

PROBLEM
- CHANGE SAMPLING RATE
- WHAT KIND OF ALIASING GETS INTRODUCED?

SOLUTION

UP

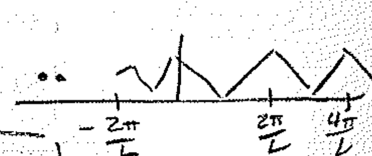
$$x[n] \longrightarrow z[n] = \begin{cases} x[m], & n = mL \\ \text{INTERPOLATED}, & \text{OTHERWISE} \end{cases}$$

IS THE SAME AS

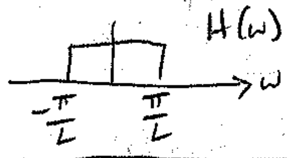
$$x[n] \longrightarrow \boxed{\uparrow L} \xrightarrow{y[n]} \boxed{H(\omega)} \xrightarrow{} z[n]$$

FOR WHICH

$$Y(\omega) = X(\omega L)$$



SO ALIASING IS ELIMINATED ONLY IF



$$H(\omega) \longleftrightarrow h[n] = \text{sinc}\left(\frac{\pi n}{L}\right)$$

DOWN

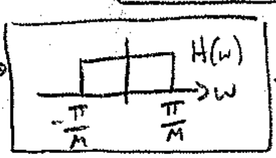
$$y[n] \longrightarrow \boxed{\downarrow M} \longrightarrow z[n] = y[nM]$$

HAS THE TRANSFORM

$$Z(\omega) = \frac{1}{M} \sum_{k=0}^{M-1} Y\left(\frac{\omega + 2\pi k}{M}\right)$$

SO ALIASING AVOIDED IFF

$x[n] \longrightarrow$



$\xrightarrow{y[n]} \boxed{\downarrow M} \longrightarrow z[n]$

8 SAMPLING

PROBLEM $\quad y(t) \longrightarrow z[n] = y(nT)$

OR

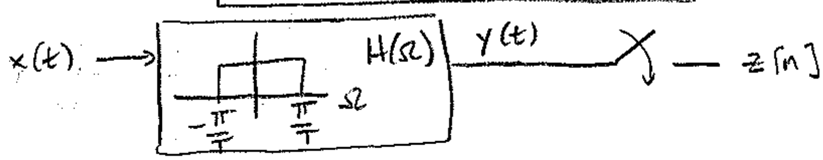
$$z[n] \longrightarrow x(t) = \sum_{n=-\infty}^{\infty} z[n]\, g(t-nT$$

WHAT KIND OF ALIASING IS INTRODUCED?

SOLUTION

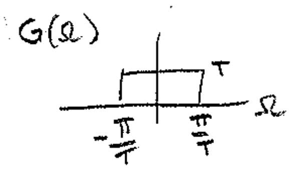
A/D $\quad Z(\omega) = \frac{1}{T} \sum_{k=-\infty}^{\infty} Y\left(\frac{\omega+2\pi k}{T}\right)$

$\boxed{\text{SO ALIASING AVOIDED ONLY IF}}$



D/A $\quad X(\Omega) = Z\left(\frac{\omega}{T}\right) G(\Omega)$

$\boxed{\text{SO ALIASING AVOIDED ONLY IF}}$



$\longleftrightarrow \quad g(t) = \mathrm{sinc}\left(\frac{\pi t}{T}\right)$

9 LPC

SYNTHESIS MODEL $\quad s[n] = e[n] + a_1 s[n-1] + \cdots + a_p s[n-p]$

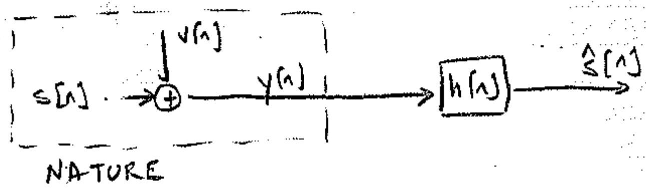
ANALYSIS MODEL $\quad e[n] = s[n] - a_1 s[n-1] - \cdots - a_p s[n-p$

PROBLEM $\quad \vec{a} = \begin{bmatrix} a_1 \\ \vdots \\ a_p \end{bmatrix} = \mathrm{argmin}\ E\left[e^2[n]\right]$

SOLUTION $\quad \vec{a} = R^{-1}\vec{r}$, E.G. IF $p=1$, $a_1 = \frac{R_{ss}[1]}{R_{ss}[0]}$

10 WIENER FILTER

PROBLEM



DESIGN $h[n]$ TO MINIMIZE $E\left[\,|\hat{s}[n] - s[n]|^2\right]$

SOLUTION

$$H(\omega) = \frac{P_{sy}(\omega)}{P_{yy}(\omega)} = \frac{P_{ss}(\omega)}{P_{ss}(\omega) + P_{vv}(\omega)}$$

IF $s[n]$ & $v[n]$ ARE UNCORRELATED AND ZERO-MEAN SIGNALS

OTHER FILTERS TO KNOW

- NOTCH FILTER
- WINDOWED FIR