UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 410 Digital Signal Processing

Quiz 3

Professors Bresler, Hasegawa-Johnson — Wednesday October 6th, 2010

Name: **Section: 1pm (E) or 3pm(G)**

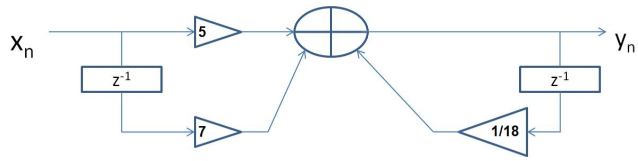
Instructions

- You may not use any calculators, cell phones, earphones, or any other forms of electronics on this quiz.
- Show all your work to receive full credit for your answers.
- When you are asked to "calculate", "determine", or "find", this means providing closed-form expressions (i.e., without summation or integration signs).
- Neatness counts. If we are unable to read your work, we cannot grade it.
- Turn in your entire booklet once you are finished. No extra booklet or papers will be considered.
- The asterisk mark ($*$) denotes convolution.

Problem	Points
1	
2	
3	
4	
Total	

Problem 1 (30 points)

Consider the causal, linear shift-invariant (LSI) system (with zero initial conditions) described in the following diagram:



(a) (5 points) Determine the linear constant coefficient difference equation relating the input $x[n]$ to the output $y[n]$ and express it in the standard form, i.e.,

$$y[n] = \sum_k \beta_k x[n-k] + \sum_j \alpha_j y[n-k]$$

(b) (10 points) Find the unit-pulse response of the system $h[n]$ using the z-transform method.

(c) (10 points) Find the zero-state response of the system when $x[n] = \left(\frac{1}{9}\right)^n u[n-3]$

(d) (5 points) Find the zero-state response of the system when $x[n] = \left(\frac{1}{9}\right)^n u[n-5]$

Problem 2 (25 points)

For the following z-transforms:

(i) Sketch the zero-pole plot.

(ii) Assume that the sequences are right-sided (i.e., causal) and sketch the region of convergence (ROC)

(iii) Determine whether the DTFT of the sequences exists.

(iv) Compute the inverse z-transform corresponding to the determined ROC

(a) (7 points) $X_1(z) = \dfrac{z^{-2}}{z + \sqrt{3}e^{j\frac{\pi}{3}}}$

(b) (9 points) $X_2(z) = \dfrac{1 + \frac{1}{3}z^{-1}}{\frac{1}{15} - \frac{8}{15}z^{-1} + \frac{1}{15}z^{-2}}$

(c) (9 points) $X_3(z) = \dfrac{-1 + z^{-1}}{(-4 + 3z^{-1} + z^{-2})}$

Problem 3 (25 points)

For each of the following sequences:

(i) Determine the z-transform, $X(z) = \sum_{n=0}^{\infty} x[n]z^{-n}$, if it exists.

(ii) Include with your answer the region of convergence of the z-transform in the z-plane.

(iii) Specify whether or not the DTFT of the sequence, $X_d(\omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$, exists.

(a) (10 points) $x[n] = (3)^n u[n-4] + (\frac{1}{8})^{n-4} u[n-2]$

(b) (10 points) $x[n] = (0.2)^{n-1}(u[n-1] - u[n-6])$

(c) (5 points) $x[n] = \sin(\pi n) - \cos(\pi n)$

Problem 4 (20 points)

Determine whether the following systems characterized by the following relations are, with respect to the input,

(i) linear or non-linear (ii) causal or non-causal (iii) shift-invariant or shift-varying

Assume that the input is zero before $n = 0$ and that the initial conditions of the systems are all set to zero. **Justify** your answers with proofs or counter-examples.

(a) (5 points) $y[n] = 2y[n-4] + 3x[n] + 7x[n-1]$

(b) (5 points) $y[n+1] = n^2 x[|n|] - 2\cos(\pi n)\, y[n]$

(c) (10 points) $y[n-1] = \sum_{k=-\infty}^{\infty} x[n-k] \left(\frac{1}{5}\right)^k u[k]$