

UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN  
Department of Electrical and Computer Engineering

ECE 498MH PRINCIPLES OF SIGNAL ANALYSIS  
Fall 2013

**MIDTERM EXAM 2 SOLUTIONS**

Wednesday, November 6, 2013

- This is a **CLOSED BOOK** exam.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must **SHOW YOUR WORK** to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: \_\_\_\_\_

### Useful Angles

$\theta$	$\cos \theta$	$\sin \theta$	$e^{j\theta}$
0	1	0	1
$\pi/6$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/2 + j/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	$\sqrt{2}/2 + j\sqrt{2}/2$
$\pi/3$	$1/2$	$\sqrt{3}/2$	$1/2 + j\sqrt{3}/2$
$\pi/2$	0	1	$j$
$\pi$	-1	0	-1
$3\pi/2$	1	-1	$-j$
$2\pi$	1	0	1

### Useful DTFTs

$$\begin{aligned}
 x[n] = a^n u[n] &\leftrightarrow X(\omega) = \frac{1}{1 - az^{-1}} \\
 x[n] = \delta[n - k] &\leftrightarrow X(\omega) = e^{-j\omega k} \\
 x[n] = e^{j\theta n} &\leftrightarrow X(\omega) = 2\pi\delta(\omega - \theta) \\
 x[n] = \left(\frac{\omega_c}{\pi}\right) \text{sinc}(\omega_c n) &\leftrightarrow X(\omega) = \begin{cases} 1 & |\omega| < \omega_c \\ 0 & \text{otherwise} \end{cases} \\
 x[n] = \begin{cases} 1 & |n| \leq M \\ 0 & \text{otherwise} \end{cases} &\leftrightarrow X(\omega) = \frac{\sin(\omega(2M + 1)/2)}{\sin(\omega/2)}
 \end{aligned}$$

**Problem 1 (20 points)**

A signal is corrupted with narrowband noise at  $\omega_n = \frac{\pi}{6}$  radians/sample. To remove the noise, you create a notch filter with a pole magnitude of  $r = 0.9$ . The filter system function is

$$H(z) = \frac{(1 - r_1 z^{-1})(1 - r_2 z^{-1})}{(1 - p_1 z^{-1})(1 - p_2 z^{-1})}$$

Specify the poles and zeros in magnitude/phase form (e.g.,  $p_1 = m e^{j\theta}$ , but you should specify the numerical value of  $m$  and the numerical value of  $\theta$ ).

$$r_1 = \boxed{e^{j\pi/6}}$$

$$r_2 = \boxed{e^{-j\pi/6}}$$

$$p_1 = \boxed{0.9e^{j\pi/6}}$$

$$p_2 = \boxed{0.9e^{-j\pi/6}}$$

**Problem 2 (20 points)**

Suppose you want to implement a filter with the following frequency response:

$$H(\omega) = \frac{(1 - 0.5e^{j\pi/4}e^{-j\omega})(1 - 0.5e^{-j\pi/4}e^{-j\omega})}{(1 + 0.5e^{j\pi/4}e^{-j\omega})(1 + 0.5e^{-j\pi/4}e^{-j\omega})}$$

You realize that you can get this frequency response by writing one line of code in matlab, implementing the following equation:

$$y[n] = x[n] + b_1x[n - 1] + b_2x[n - 2] + a_1y[n - 1] + a_2y[n - 2]$$

Find the filter coefficients:

$$a_1 = \boxed{-\frac{\sqrt{2}}{2}}$$

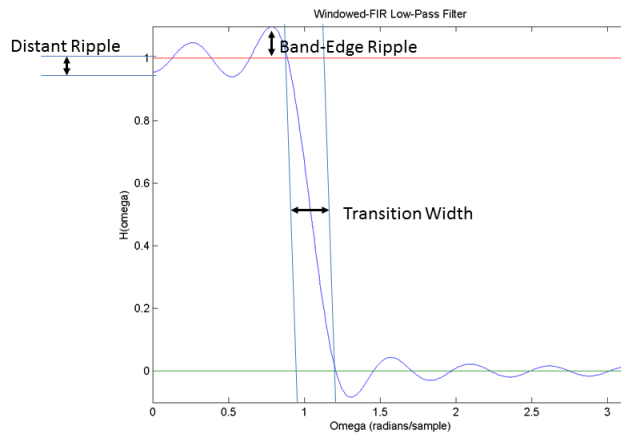
$$a_2 = \boxed{-\frac{1}{4}}$$

$$b_1 = \boxed{-\frac{\sqrt{2}}{2}}$$

$$b_2 = \boxed{\frac{1}{4}}$$

**Problem 3 (20 points)**

An FIR filter will be created as  $h[n] = w[n]d[n]$ , for  $D(\omega) = \begin{cases} 1 & |\omega| < \frac{\pi}{3} \\ 0 & \text{else} \end{cases}$ . You are considering three possible windows: rectangular, Hamming, and Hann, each of length  $2M + 1 = 23$  samples. These three windows are differentiated by three typical measurements, depicted in the figure below: transition width, band-edge ripple, and distant ripple.



Your evil cousin Morwen has calculated transition width, band-edge ripple, and distant ripple for the rectangular, Hamming, and Hann windows. Out of spite, however, she has mixed them up, so that you don't know which one goes with which window; all you know is that, for each of the three windows,

- Transition Width is either  $\frac{2\pi}{23}$  or  $\frac{4\pi}{23}$
- Band-Edge Ripple is either 0.0033 or 0.0062 or 0.099
- Distant Ripple is either 0.0006 or 0.0031 or 0.045

Sort the values above into the table below.

Window	Rectangular	Hamming	Hann
Transition Width	$\frac{2\pi}{23}$	$\frac{4\pi}{23}$	$\frac{4\pi}{23}$
Band-Edge Ripple	0.099	0.0033	0.0062
Distant Ripple	0.045	0.0031	0.0006

**Problem 4 (20 points)**

Consider

$$x[n] \rightarrow \boxed{\text{System}} \rightarrow y[n]$$

where the system is defined by the relationship  $y[n] = w[n]x[n]$  for

$$w[n] = \begin{cases} 1 & |n| \leq 7 \\ 0 & \text{otherwise} \end{cases}$$

Prove that this system is **not** time-invariant.

**Proof**

The fastest way to prove this is by counter-example. For example,

$$\begin{aligned} x_1[n] = \delta[n] &\rightarrow y_1[n] = \delta[n] \\ x_2[n] = x_1[n-8] &\rightarrow y_2[n] = 0 \neq y_1[n-8] \end{aligned}$$

**Problem 5 (20 points)**

Consider the signal  $y[n]$  defined by

$$y[n] = \begin{cases} e^{j0.42\pi n} & |n| \leq 11 \\ 0 & \text{otherwise} \end{cases}$$

Use the windowing property ( $w[n]x[n] \leftrightarrow \frac{1}{2\pi}W(\omega) * X(\omega)$ ) to find  $Y(\omega)$ , the DTFT of  $y[n]$ .

**Solution**

$$Y(\omega) = \frac{\sin\left(\frac{23}{2}(\omega - 0.42\pi)\right)}{\sin\left(\frac{1}{2}(\omega - 0.42\pi)\right)}$$