UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 498MH Signal and Image Analysis

Homework 2
Fall 2014

Assigned: Thursday, September 4, 2014 Due: **Wednesday**, September 10, 2014

Reading: Jason Starck, *All About Circuits* Chapter 7: Mixed-Frequency AC Signals,
`http://www.allaboutcircuits.com/vol_2/chpt_7/`

1 Complex Numbers

Do **one** of the following three problems.

Problem 2.1.1

$$x[n] = \cos\left(\frac{2\pi n}{10}\right) + 3\sin\left(\frac{2\pi n}{10}\right)$$

(a) This signal can be written as

$$x[n] = \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)e^{j\omega n} + \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)^* e^{-j\omega n}$$

Find A, B, θ, ϕ, and ω (express them as integers, or as irreducible fractions. Don't use a calculator).

(b) This signal can be written

$$x[n] = M\cos(\omega n + \psi)$$

Find M and ψ. Do **not** use a calculator. Instead, write M in the form $M = \sqrt{N}$ for some integer N. Write ψ in the form of either $\psi = \text{atan}(\alpha)$, or $\psi = -\text{atan}(\alpha)$, or $\psi = \pi - \text{atan}(\alpha)$, or $\psi = -\pi + \text{atan}(\alpha)$, where α is an irreducible **positive** fraction (irreducible means that the numerator and denominator are integers with no common divisors; positive means that it is greater than zero).

Problem 2.1.2

$$x[n] = 5\cos\left(\frac{2\pi n}{15}\right) + 2\sin\left(\frac{2\pi n}{15}\right)$$

(a) This signal can be written as

$$x[n] = \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)e^{j\omega n} + \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)^* e^{-j\omega n}$$

Find A, B, θ, ϕ, and ω (express them as integers, or as irreducible fractions. Don't use a calculator).

(b) This signal can be written

$$x[n] = M\cos(\omega n + \psi)$$

Find M and ψ. Do **not** use a calculator. Instead, write M in the form $M = \sqrt{N}$ for some integer N. Write ψ in the form of either $\psi = \text{atan}(\alpha)$, or $\psi = -\text{atan}(\alpha)$, or $\psi = \pi - \text{atan}(\alpha)$, or $\psi = -\pi + \text{atan}(\alpha)$, where α is an irreducible **positive** fraction (irreducible means that the numerator and denominator are integers with no common divisors; positive means that it is greater than zero).

Problem 2.1.3

$$x[n] = 3\cos\left(\frac{2\pi n}{7}\right) + 10\sin\left(\frac{2\pi n}{7}\right)$$

(a) This signal can be written as

$$x[n] = \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)e^{j\omega n} + \frac{1}{2}\left(Ae^{j\theta} + Be^{j\phi}\right)^* e^{-j\omega n}$$

Find A, B, θ, ϕ, and ω (express them as integers, or as irreducible fractions. Don't use a calculator).

(b) This signal can be written

$$x[n] = M\cos(\omega n + \psi)$$

Find M and ψ. Do **not** use a calculator. Instead, write M in the form $M = \sqrt{N}$ for some integer N. Write ψ in the form of either $\psi = \text{atan}(\alpha)$, or $\psi = -\text{atan}(\alpha)$, or $\psi = \pi - \text{atan}(\alpha)$, or $\psi = -\pi + \text{atan}(\alpha)$, where α is an irreducible **positive** fraction (irreducible means that the numerator and denominator are integers with no common divisors; positive means that it is greater than zero).

2 Fourier Series

Do **one** of the following three problems.

Problem 2.2.1

Consider the signal

$$x(t) = |\cos(2\pi t)|$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: notice that $|\cos(2\pi t)|$ is sometimes equal to $\cos(2\pi t)$, and sometimes equal to $-\cos(2\pi t)$, so if you choose the right period of time over which to integrate, you might be able to get rid of the absolute value signs.
- Hint #2: use the relationship $\cos(2\pi t) = \frac{1}{2}(e^{j2\pi t} + e^{-j2\pi t})$ so that you can integrate exponentials instead of integrating cosines.
- Hint #3:

$$\begin{aligned}\int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right)\end{aligned}$$

Problem 2.2.2

Consider the signal

$$x(t) = 1 - |\sin(200\pi t)|$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: notice that $|\sin(200\pi t)|$ is sometimes equal to $\cos(2\pi t)$, and sometimes equal to $-\sin(200\pi t)$, so if you choose the right period of time over which to integrate, you might be able to get rid of the absolute value signs.
- Hint #2: use the relationship $\sin(200\pi t) = \frac{1}{2j}(e^{j200\pi t} - e^{-j200\pi t})$ so that you can integrate exponentials instead of integrating cosines.
- Hint #3:

$$\begin{aligned} \int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right) \end{aligned}$$

Problem 2.2.3

Consider the signal

$$x(t) = \begin{cases} 1 - e^{-100|t|} & -0.01 \leq t \leq 0.01 \\ x(t - 0.02) & \text{otherwise} \end{cases}$$

(a) Sketch $x(t)$.

(b) What is its period, T_0? What is its fundamental frequency, ω_0?

(c) Find the Fourier series coefficients.

- Hint #1: for any time points $a \leq b \leq c$,

$$\int_a^c x(t)dt = \int_a^b x(t)dt + \int_b^c x(t)dt$$

- Hint #2: notice that $e^{-100|t|}$ is sometimes equal to e^{-100t}, and sometimes equal to e^{100t}, so if you divide the integral as shown in Hint #1, then you won't have to use the absolute value sign any more. (And notice that $e^{-100t}e^{jk\omega_0 t} = e^{(-100+jk\omega_0)t}$).
- Hint #3:

$$\begin{aligned} \int_c^d (e^{at} + e^{bt})dt &= \left[\frac{1}{a}e^{at} + \frac{1}{b}e^{bt}\right]_c^d \\ &= \left(\frac{1}{a}e^{ad} + \frac{1}{b}e^{bd}\right) - \left(\frac{1}{a}e^{ac} + \frac{1}{b}e^{bc}\right) \end{aligned}$$