ECE 401 Signal and Image Analysis
Homework 6

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Monday, 11/16/2020; Due: Monday, 11/30/2020
Reading: DSP First, Section 7.2.5

Problem 6.1

Suppose that you have a zero-mean unit-variance random signal, $x[n]$, whose samples are perfectly periodic ($x[n+P] = x[n]$ for all n), but are otherwise completely unpredictable ($x[n+k]$ and $x[n]$ are independent for $1 \leq k < P$). What is the expected autocorrelation of this signal?

Problem 6.2

Suppose that $y[n] = x[n] * h[n]$, where $x[n]$ is zero-mean white noise with variance σ^2, and $h[n] = a^n u[n]$ for some real constant $0 < a < 1$. What is $E\left[r_{yy}[n]\right]$, the autocorrelation of $y[n]$? What is the average signal power, $E\left[r_{yy}[0]\right]$?

Problem 6.3

Use Parseval's theorem (any of its forms) to evaluate the following integral:

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \frac{1}{|1 - ae^{-j\omega}|^2} d\omega$$

Problem 6.4

Suppose that $x[n]$ is a zero-mean Gaussian noise signal with the following DTFT power spectrum:

$$E\left[R_{xx}(\omega)\right] = \begin{cases} \sigma^2 & |\omega| < \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |\omega| < \pi \end{cases}$$

What is the expected autocorrelation, $E\left[r_{xx}[n]\right]$?