ECE 401 Signal and Image Analysis
Homework 1

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Tuesday, 8/25/2020; Due: Monday, 8/31/2020
Reading: *DSP First* Appendix A

Problem 1.1

Find $\angle z$ as a function of a and b.

$$z = e^{ja} + e^{jb} \tag{1.1-1}$$

Solution:

$$\angle a = \text{atan}\left(\frac{\sin(a) + \sin(b)}{\cos(a) + \cos(b)}\right)$$

Problem 1.2

Evaluate this integral:

$$\int_0^T e^{at} dt \tag{1.2-1}$$

Solution:

$$\frac{1}{a}\left(e^{aT} - 1\right)$$

Problem 1.3

In MP1, one of the filters you'll create is a local averaging filter. A local averaging filter produces an output $y[n]$, at time n, which is the average of the previous N samples of $x[m]$:

$$y[n] = \frac{1}{N} \sum_{m=n-(N-1)}^{n} x[m] \tag{1.3-1}$$

(a) First, consider what happens if $x[m]$ is a pure tone with a period of T:

$$x[m] = \cos\left(\frac{2\pi m}{T}\right)$$

Suppose that the averaging window, N, is exactly an integer multiple of T. For example, suppose that $N = 3T$. Draw a picture of $x[m]$ as a function of m, and shade in the regions that would be added together by the summation in Eq. (1.3-1) in order to compute $y[0]$. Argue based on your figure (with no calculations at all) that $y[0] = 0$.

Solution: Every period of the cosine has a positive section and a negative section. When we average these two sections, they cancel each other out.

(b) Adding up the samples of a cosine is easy when N is an integer multiple of T, but hard otherwise. It's actually much easier to add the samples of a complex exponential, because we can use the standard geometric series formula (`https://en.wikipedia.org/wiki/Geometric_series#Formula`). Use that formula to find $y[0]$ when

$$x[m] = e^{j2\pi m/T}$$

Your result should have the form $y[0] = (1 - a)/(1 - b)$ for some a and b that depend on π, N, and T, but not on m or n.

Solution:

$$y[0] = \frac{1 - e^{-j2\pi N/T}}{1 - e^{-j2\pi/T}}$$