# ECE 401 Signal and Image Analysis
# Homework 6

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Monday, 11/16/2020; Due: Monday, 11/30/2020
Reading: DSP First, Section 7.2.5

**Problem 6.1**

Suppose that you have a zero-mean unit-variance random signal, $x[n]$, whose samples are perfectly periodic ($x[n+P] = x[n]$ for all $n$), but are otherwise completely unpredictable ($x[n+k]$ and $x[n]$ are independent for $1 \le k < P$). What is the expected autocorrelation of this signal?

**Solution:**

$$E\left[r_{xx}[n]\right] = \begin{cases} 1 & n = \ell P \text{ for any integer } \ell \\ 0 & \text{otherwise} \end{cases}$$

**Problem 6.2**

Suppose that $y[n] = x[n] * h[n]$, where $x[n]$ is zero-mean white noise with variance $\sigma^2$, and $h[n] = a^n u[n]$ for some real constant $0 < a < 1$. What is $E\left[r_{yy}[n]\right]$, the autocorrelation of $y[n]$? What is the average signal power, $E\left[r_{yy}[0]\right]$?

**Solution:**

$$E\left[r_{yy}[n]\right] = \sigma^2 \frac{a^{|n|}}{1-a^2}$$

The average signal power is

$$E\left[r_{yy}[0]\right] = \sigma^2 \frac{1}{1-a^2}$$

**Problem 6.3**

Use Parseval's theorem (any of its forms) to evaluate the following integral:

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} \frac{1}{|1 - ae^{-j\omega}|^2} d\omega$$

**Solution:** Any of the forms of Parseval's theorem can be used to solve this. Perhaps the simplest form is

$$\sum_{n=-\infty}^{\infty} x^2[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} |X(\omega)|^2 d\omega$$

The right-hand side is the integral we want to find. The left-hand side involves the inverse transform of $1/(1 - ae^{-j\omega})$, which is $x[n] = a^n u[n]$, therefore

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{|1-ae^{-j\omega}|^2}d\omega = \sum_{n=0}^{\infty} a^{2n} = \frac{1}{1-a^2}$$

We could also use the power form of Parseval's theorem, which says that

$$r_{xx}[0] = \frac{1}{2\pi}\int_{-\pi}^{\pi} R_{xx}(\omega)d\omega$$

In order to use this form, you'd have to realize that $\frac{1}{|1-ae^{-j\omega}|^2}$ is the Fourier transform of the autocorrelation function you found in problem 2, and therefore

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\frac{1}{|1-ae^{-j\omega}|^2}d\omega = r_{xx}[0] = \frac{1}{1-a^2}$$

**Problem 6.4**

Suppose that $x[n]$ is a zero-mean Gaussian noise signal with the following DTFT power spectrum:

$$E\left[R_{xx}(\omega)\right] = \begin{cases} \sigma^2 & |\omega| < \frac{\pi}{3} \\ 0 & \frac{\pi}{3} < |\omega| < \pi \end{cases}$$

What is the expected autocorrelation, $E\left[r_{xx}[n]\right]$?

**Solution:**

$$E\left[r_{xx}[n]\right] = \frac{\sigma^2 \sin(\pi n/3)}{\pi n}$$