UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering

ECE 417 Multimedia Signal Processing
Spring 2016

EXAM 2

Thursday, March 31, 2016

- This is a CLOSED BOOK exam. You may use one sheet (front and back) of hand-written notes.
- No calculators are permitted. You need not simplify explicit numerical expressions.
- There are a total of 100 points in the exam. Each problem specifies its point total. Plan your work accordingly.
- You must SHOW YOUR WORK to get full credit.

Problem	Score
1	
2	
3	
4	
5	
Total	

Name: ______________________________

Possibly Useful Formulas

Gaussian (Normal) Distribution A Gaussian is parameterized by $\vec{\mu}$, Σ, and $D = \dim(\vec{\mu})$ as

$$\mathcal{N}(\vec{x}|\vec{\mu},\Sigma) = \frac{1}{(2\pi)^{D/2}|\Sigma|^{1/2}} e^{-\frac{1}{2}(\vec{x}-\vec{\mu})^T\Sigma^{-1}(\vec{x}-\vec{\mu})}$$

Gaussian Mixture Model (GMM) A GMM is parameterized by c_k, $\vec{\mu}_k$, and Σ_k for $1 \le k \le K$ as

$$p_X(\vec{x}) = \sum_{k=1}^{K} c_k \mathcal{N}(\vec{x}|\vec{\mu}_k,\Sigma_k)$$

Hidden Markov Model (HMM) An HMM is parameterized by $\lambda = \{\pi_i, a_{ij}, b_j(\vec{x})\}$, where

$$\begin{aligned} \pi_i &= p(q_1 = i|\lambda), \quad 1 \le i \le N \\ a_{ij} &= p(q_{t+1} = j|q_t = i,\lambda), \quad 1 \le i,j \le N \\ b_j(\vec{x}) &= p(\vec{x}|q_t = j,\lambda), \quad 1 \le j \le N \end{aligned}$$

The acoustic model $b_j(\vec{x})$ might be GMM, for example, in which case the HMM parameters include

$$\begin{aligned} c_{jk} &= p(g_t = k|q_t = j) \\ \vec{\mu}_{jk} &= E\left[\vec{x}_t|q_t = j, g_t = k\right] \\ \Sigma_{jk} &= E\left[(\vec{x}_t - \vec{\mu}_{jk})(\vec{x}_t - \vec{\mu}_{jk})^T|q_t = j, g_t = k\right] \end{aligned}$$

Scaled Forward Algorithm

$$\begin{aligned} \hat{\alpha}_1(i) &= \pi_i b_i(\vec{x}_1), \quad 1 \le i \le N \\ g_1 &= \sum_{i=1}^{N} \hat{\alpha}_1(i) \\ \tilde{\alpha}_1(i) &= \frac{1}{g_1}\hat{\alpha}_1(i) \\ \hat{\alpha}_t(j) &= \sum_{i=1}^{N} \tilde{\alpha}_{t-1}(i) a_{ij} b_j(\vec{x}_t) \\ g_t &= \sum_{j=1}^{N} \hat{\alpha}_t(j) \\ \tilde{\alpha}_t(j) &= \frac{1}{g_t}\hat{\alpha}_t(j) \\ p(\vec{x}_1,\ldots,\vec{x}_t|\lambda) &= \prod_{t=1}^{T} g_t \end{aligned}$$