1 Exam 2 Solutions

Problem 1 (20 points)

$\theta = 4.5$

Problem 2 (20 points)

$$p_{Y|X}\left(1 \middle| \begin{bmatrix} x_{10} \\ 0 \end{bmatrix}\right) = \begin{cases} 0 & x_{10} < \frac{3}{4} \\ \frac{1}{3} & \frac{3}{4} < x_{10} < 3 \\ \frac{2}{3} & 3 < x_{10} < \frac{13}{4} \\ 1 & \frac{13}{4} < x_{10} \end{cases}$$

Problem 3 (20 points)

Modes of the distribution are at $[-2, 0]^T$ and $[2, 0]^T$. The $e^{-1/2}$ contour lines are ellipses: a 4×2 ellipse centered at $[-2, 0]^T$, and a 2×4 ellipse centered at $[2, 0]^T$. The e^{-2} contour line is the continuous outer hull of the 8×4 and 4×8 ellipses centered at the modes.

Problem 4 (20 points)

There are several possible solutions. One is

$$\begin{aligned}
p(q_1 = k, \vec{x}_1|\lambda) &= \frac{1}{N} b_k(\vec{x}_1) \\
p(q_2 = i, q_1 = k, \vec{x}_1, \vec{x}_2|\lambda) &= p(q_1 = k, \vec{x}_1|\lambda) a_{ki} b_i(\vec{x}_2) \\
p(q_t = j, q_1 = k, \vec{x}_1, \ldots, \vec{x}_t|\lambda) &= \sum_{i=1}^{N} p(q_{t-1} = i, q_1 = k, \vec{x}_1, \ldots, \vec{x}_{t-1}|\lambda) a_{ij} b_j(\vec{x}_t) \\
p(q_1 = k, \vec{x}_1, \ldots, \vec{x}_T|\lambda) &= \sum_{j=1}^{N} p(q_T = j, q_1 = k, \vec{x}_1, \ldots, \vec{x}_T|\lambda)
\end{aligned}$$

$$p(q_1 = k|\vec{x}_1, \ldots, \vec{x}_T, \lambda) = \frac{p(q_1 = k, \vec{x}_1, \ldots, \vec{x}_T|\lambda)}{\sum_{\ell=1}^{N} p(q_1 = \ell, \vec{x}_1, \ldots, \vec{x}_T|\lambda)}$$

Problem 5 (20 points)

$$p(q_{t-1} = i, q_t = j, \vec{x}_{t-1}, \vec{x}_t|\vec{x}_1, \ldots, \vec{x}_{t-2}, \lambda) = \hat{\alpha}_{t-1}(i) a_{ij} b_j(\vec{x}_t)$$