UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 12 Sample Problems

Problem 12.1

As you know by now, zero-padding an image prior to filtering results in weird artifacts (edge effects), but for this problem, let's assume that zero-padding works, just because it makes the math easier. So, assume that you have an image $x[n_1, n_2]$ that is well-defined for $(-\infty < n_1 < \infty, -\infty < n_2 < \infty)$, but whose pixels are all zero except in the range $(0 \le n_1 < N_1, 0 \le n_2 < N_2)$.

Suppose that you want to downsample the image, to create an image $y[n_1, n_2]$ of size $\left(\frac{N_1}{5}, \frac{N_2}{3}\right)$. To do so, you decide that you'll first filter it using an ideal lowpass filter, computing $f[n_1, n_2] = x[n_1, n_2] ** h[n_1, n_2]$ where

$$H(\omega_1, \omega_2) = \begin{cases} 1 & \left(-\frac{\pi}{5} \le \omega_1 \le \frac{\pi}{5}, -\frac{\pi}{3} \le \omega_2 \le \frac{\pi}{3}\right) \\ 0 & \text{otherwise} \end{cases} \qquad (12.1\text{-}1)$$

and then, once that is done, you will downsample, as

$$y[n_1, n_2] = f[5n_1, 3n_2]$$

1. What are the coefficients of the filter $h[n_1, n_2]$ whose frequency response is given in Eq. 12.1-1?
2. The standard way to implement 2D convolution is

   $$f[n_1, n_2] = \sum_{m_1=\infty}^{\infty} \sum_{m_2=-\infty}^{\infty} x[m_1, m_2] h[n_1 - m_1, n_2 - m_2] \qquad (12.1\text{-}2)$$

   Suppose your goal is to compute the "full" convolution response. How many multiply-add operations (of terms other than zero) are required to perform this operation using Eq. 12.1-2?
3. Use the method of separable filters to devise an algorithm that generates $f[n_1, n_2]$ using no more than $\mathcal{O}\left\{N_1 N_2 (N_1 + N_2)\right\}$ multiply-add operations.

Problem 12.2

The reason that sinc-squared interpolation is sometimes better than sinc-interpolation: natural images tend to have $1/f$ spectra. This means that the spectrum of a natural image is often of the form $X(e^{j\omega}) = \frac{1}{|\omega|}$ over a wide range of frequencies, from a low frequency equal to the low-frequency cutoff of the recording microphone (call that ω_L, maybe) up to Nyquist.

Suppose that $u[n]$ is a signal with a $1/f$ spectrum. Suppose you lowpass filter with an ideal $\pi/2$ lowpass filter to produce $v[n]$, then downsample by a factor of 2 to produce $x[n]$, then upsample by 2 to produce $y[n]$, then filter with some interpolating filter $h[n]$ to produce the output $z[n]$.

1. Suppose that $h[n]$ is an ideal lowpass filter,

   $$h_a[n] = \frac{\sin(\pi n/2)}{\pi n/2}$$

   What is the spectrum of $z[n]$? How does it compare to the spectrum of $u[n]$?

2. Now suppose that $h[n]$ is a sinc-squared,

$$h_b[n] = \left(\frac{\sin(\pi n/2)}{\pi n/2}\right)^2$$

What is the spectrum of $z[n]$? How does it compare to the spectrum of $u[n]$?