UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering
ECE 417 Multimedia Signal Processing

Lecture 3 Sample Problems

Problem 3.1

$x[n]$ is zero-mean white Gaussian random noise. What is the probability that the first ten samples, $x[0]$ through $x[9]$, are all positive?

Problem 3.2

Non-zero-mean white Gaussian noise has the following properties:

$$\begin{aligned} x[n] &\sim \mathcal{N}(\mu[n], \sigma^2) \\ E\left\{x[n]x[m]\right\} &= \mu[n]\mu[m] \end{aligned}$$

Let $X[k]$ be the DFT of $x[n]$. Find $E\left\{X[k]\right\}$ in terms of $\mu[n]$ and/or σ^2.

Problem 3.3

White noise is called "white" because it has a flat spectrum, like white light. Pink noise is called "pink" because it has a mildly lowpass spectrum, like pink light. For example, suppose $x[n] \sim \mathcal{N}(0, \sigma^2)$ but $E\left\{x[n]x[m]\right\} = \sigma^2\rho^{|n-m|}$, i.e., $x[n]$ and $x[m]$ are correlated with a correlation coefficient $\rho^{|n-m|}$. You may assume $|\rho| < 1$. Suppose that $X[k]$ is the DFT of $x[n]$. Find $E\left\{|X[k]|^2\right\}$. Hint: use $\sum_{n=0}^{N-1} a^n = \frac{1-a^N}{1-a}$, but don't try to convert your answer into a dsinc function.

Problem 3.4

There are three types of autocorrelation you need to know about in this course. The statistical autocorrelation function is

$$R_{xx}[n] = E\left\{x[m]x[m-n]\right\}$$

The linear autocorrelation function is

$$r_{xx}[n] = x[n] * x[-n] = \sum_{m=-\infty}^{\infty} x[m]x[m-n]$$

The circular autocorrelation function is

$$\tilde{r}_{xx}[n] = x[n] \circledast x[-n] = \sum_{m=0}^{N-1} x\left[m\right] x\left[< m-n >_N\right]$$

Show that linear autocorrelation and circular autocorrelation are the same, for $0 \le n \le \frac{N}{2} - 1$, if $x[n]$ is nonzero only in the domain $n \in \left[0, \frac{N}{2} - 1\right]$.