ECE 417 Multimedia Signal Processing
Homework 6

UNIVERSITY OF ILLINOIS
Department of Electrical and Computer Engineering

Assigned: Monday, 11/16/2020; Due: Wednesday, 12/2/2020

Problem 6.1

Suppose you have a recurrent neural network with input $x[n]$, target $y[n]$, output $\hat{y}[n]$, and error metric

$$\mathcal{E} = -\frac{1}{N}\sum_{n=0}^{N-1}\left(y[n]\ln\hat{y}[n] + (1-y[n])\ln(1-\hat{y}[n])\right)$$

where

$$\begin{aligned}\hat{y}[n] &= \sigma(e[n]),\\ e[n] &= x[n] + \sum_{m=1}^{M-1} w[m]\hat{y}[n-m],\end{aligned}$$

and where $\sigma(\cdot)$ is the logistic sigmoid. Write $d\mathcal{E}/dw[3]$ in terms of the signals $y[n]$ and $\hat{y}[m]$. You can invent auxiliary signals such as $\dot{\sigma}[n]$, $\epsilon[n]$, or $\delta[n]$ if you wish, but you need to define them clearly. You may assume that $\hat{y}[n] = 0$ for $n < 0$.

Problem 6.2

Suppose that

$$\begin{aligned}h_0 &= x^3\\ h_1 &= \cos(x) + \sin(h_0)\\ \hat{y} &= \frac{1}{2}\left(h_1^2 + h_0^2\right)\end{aligned}$$

What is $d\hat{y}/dx$? Express your answer as a function of x only, without the variables h_0 or h_1 in your answer.

Problem 6.3

Consider a one-gate recurrent neural net, defined as follows:

$$\begin{aligned}c[n] &= c[n-1] + w_c x[n] + u_c h[n-1] + b_c\\ h[n] &= o[n]c[n]\\ o[n] &= \sigma(w_o x[n] + u_o h[n-1] + b_o)\end{aligned}$$

where $\sigma(\cdot)$ is the logistic sigmoid, $x[n]$ is the network input, $c[n]$ is the cell, $o[n]$ is the output gate, and and $h[n]$ is the output. Suppose that you've already completed synchronous back-prop, which has given you the following quantity:

$$\epsilon[n] = \frac{\partial E}{\partial h[n]}$$

Find asynchronous back-prop formulas, i.e., find formulas for the following quantities, in terms of one another, and/or in terms of the other quantities defined above:

$$\delta_h[n] = \frac{dE}{dh[n]}$$
$$\delta_o[n] = \frac{dE}{do[n]}$$
$$\delta_c[n] = \frac{dE}{dc[n]}$$

Problem 6.4

Using the CReLU nonlinearity for both σ_h and σ_g in an LSTM, choose weights and biases,

$$\{b_c, u_c, w_c, b_f, u_f, w_f, b_i, u_i, w_i, b_o, u_o, w_o\},$$

that will cause an LSTM to count the number of nonzero inputs, and output the tally only when the input is zero:

$$h[n] = \begin{cases} \sum_{m=0}^{n} \mathbf{1}[x[m] \geq 1] & x[n] = 0 \\ 0 & x[n] \geq 1 \end{cases}$$

where $\mathbf{1}[\cdot]$ is the unit indicator function, and you may assume that $x[n]$ is always a non-negative integer.