

# Collab Worksheet 7

CS440/ECE448, Spring 2021

Week of 3/26 - 3/31, 2021

## Question 1

Ron the Con is a skilled con artist who works Green Street. After showing the audience a fair coin, Ron will either continue to use the fair coin (with probability  $1 - P$ ) or secretly exchange it for an unfair coin (with probability  $P$ ). If he flips the fair coin, it comes up heads with probability 50%. If he flips the unfair coin, it comes up heads with probability 100%. After every flip, he either continues to use the same coin (with probability  $1 - P$ ), or he secretly switches coins (with probability  $P$ ).

- (a) (1 point) This process can be represented as a hidden Markov model, where the state variable is  $Y_t = 1$  for the unfair coin,  $Y_t = 0$  for the fair coin, and the evidence variable is  $X_t = 1$  for heads,  $X_t = 0$  for tails. Write the conditional probability table  $p(X_t|Y_t)$ .

- (b) (2 points) For this part of the problem only, assume that after every flip, Ron either continues to use the same coin (with probability  $1 - P$ ), or he secretly switches coins (with probability  $P$ ). Write the conditional probability table  $p(Y_t|Y_{t-1})$ .

- (c) (2 points) For this part of the problem only, assume that after every flip, Ron NEVER changes the coin. In other words, with probability  $P$ , he chooses the unfair coin on the first flip, and then continues to use the unfair coin for every flip thereafter; with probability  $1 - P$ , he chooses the fair coin on the first flip, then he continues to use the fair coin for every flip thereafter. In terms of  $P$ , how many heads in a row would you need to see before you conclude that the coin is unfair?

### Question 2

Consider a hidden Markov model (HMM) whose hidden variable denotes part of speech (POS),  $Y_t \in \{N, V\}$  where  $N$  =noun,  $V$  =verb, the initial state probability is  $P(Y_1 = N) = 0.8$ , and the transition probabilities are  $P(Y_t = N|Y_{t-1} = N) = 0.1$  and  $P(Y_t = V|Y_{t-1} = V) = 0.1$ . Suppose we have the observation probability matrix given in the following table:

$X_t$	rose	bill	likes
$P(X_t Y_t = N)$	0.4	0.4	0.2
$P(X_t Y_t = V)$	0.2	0.2	0.6

You are given the sentence “bill rose.” You want to figure out whether each of these two words, “bill” and “rose”, is being used as a noun or a verb.

- (a) List the four possible combinations of  $(Y_1, Y_2)$ . For each possible combination, give  $P(Y_1, X_1 = \text{bill}, Y_2, X_2 = \text{rose})$ . Give your results as mathematical expressions in terms of constants; do not simplify.

- (b) Let  $\mathcal{D}$  be the event  $(X_1 = \text{bill}, X_2 = \text{rose})$ . Find  $P(Y_2 = V|\mathcal{D})$ .

- (c) Use the Viterbi algorithm to find the most likely state sequence for this sentence.

