UNIVERSITY OF ILLINOIS AT URBANA-CHAMPAIGN
Department of Electrical and Computer Engineering
Instructor: Mark Hasegawa-Johnson

ECE 537 Speech Processing

Problem Set 7

Fall 2009

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Reading for problem set 7: Allen, *Nonlinear Cochlear Signal Processing...*

Problem 7.1

Describe briefly the function of each of the following organs.

(a) Pinna Solution: The pinna is a direction-dependent filter, allowing us to better localize sounds.

(b) Concha Solution: The concha protects the eardrum from injury. It also acts as a quarter-wave resonator, adding about 8dB to the sensation level of sounds in the 2-4kHz range.

(c) Ossicles Solution: The ossicles act as a nonlinear impedance matcher. They increase the effectiveness with which waves in the air of the concha are transmitted to the salt water of the cochlea. Supporting ligaments loosen the ossicles in presence of high-intensity sounds, reducing the effectiveness of sound transmission to the cochlea.

(d) Oval window Solution: The stapes move the oval window, transmitting sound to the cochlea.

(e) Basilar membrane Solution: The basilar membrane acts like a bank of mechanical bandpass filters.

(f) Tectorial membrane Solution: Interaction between the basilar membrane and the tectorial membrane, mediated by the outer hair cells, improves the frequency selectivity of the former.

(g) Reticular lamina Solution: The reticular lamina electrically isolates the scala media from the scala vestibuli, so that its resting voltage can be 50-70mV higher. Since the scala vestibuli is connected to the scala tympani, and hence to the organ of Corti, this electrical isolation allows the hair cell resting voltage to be 50-70mV below that of the scala media.

(h) Inner hair cell Solution: The IHC is a mechano-electric transducer. Movement of the basilar membrane opens pores in the stereocilia. Potassium ions flow through the pores, raising the voltage inside the hair cell above the voltage of the surrounding organ of Corti. Raised voltage inside the hair cell causes it to release neurotransmitter, which binds to nearby auditory nerve fibers.

(i) Column of corti Solution: The column of Corti is a structural support, so that the hair cells don't get squashed.

(j) Outer hair cell Solution: The OHC is also a mechano-electric transducer, but it probably acts most of the time in reverse, as a kind of electro-mechanical transducer. Efferent neurons change the membrane voltage of the OHC, causing it to stiffen or slacken as necessary in response to efferent input. This selective modification of the BM stiffness probably increases sensitivity (and frequency selectivity) at low SPL, while allowing the membrane to be less sensitive (but hence less frequency-selective) at high SPL.

Problem 7.2

What is the power spectrum of a sinusoid? Be sure to label units, and demonstrate Parseval's theorem. Solution:

$$x(t) = A\cos(\Omega_0 t + \theta), \quad R_x(\tau) = \frac{A^2}{2}\cos(\Omega_0 t)$$

$$S_x(\Omega) = \frac{\pi A^2}{2}\left(\delta(\Omega - \Omega_0) + \delta(\Omega + \Omega_0)\right)$$

$$\frac{1}{2\pi}\int_{-\infty}^{\infty} S_x(\Omega) d\Omega = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)dt = \frac{A^2}{2}$$

Problem 7.3

(a) $v(t)$ is continuous-time wideband noise, with bandwidth B Hz and noise power density N_0 Watts/m^2/Hz. Sketch its power spectrum, $S_X(\Omega)$. Compute its autocorrelation, $R_X(\tau)$. What is the noise power?

Solution: $S_v(\Omega)$ is flat at $N_0/2\pi$ W/m^2/(radian/sec) between $-2\pi B$ and $2\pi B$ radians/second.

$$R_v(\tau) = \frac{BN_0}{\pi}\text{sinc}(2\pi B\tau)$$

The noise power is $R_v(0) = BN_0/\pi$.

(b) $w(t) = h(t) * v(t)$ is a narrowband noise source, where $h(t)$ is an ideal bandpass filter with center frequency 2kHz and bandwidth 200Hz. Compute the power spectrum, autocorrelation, and signal power of $w(t)$.

Solution: $S_w(\Omega) = |H(\Omega)|^2 S_v(\Omega)$ is flat at $N_0/2\pi$ for $2\pi 1900 \leq |\Omega| \leq 2\pi 2100$, and zero elsewhere.

$$R_w(\tau) = \left(\frac{2\pi 2100(N_0/2\pi)}{\pi}\right)\text{sinc}(2\pi 2100\tau) - \left(\frac{2\pi 1900(N_0/2\pi)}{\pi}\right)\text{sinc}(2\pi 1900\tau)$$

A different derivation gives a form which is the same, though not obviously so:

$$R_w(\tau) = \left(\frac{2\pi 100}{\pi}\right)\left(\frac{N_0}{\pi}\right)\text{sinc}(2\pi 100\tau)\cos(2\pi 2000\tau)$$

The total power is $R_w(0) = 200N_0/\pi$.

(c) $v[n]$ is discrete-time white noise, with variance σ_x^2. Compute the autocorrelation, power spectrum, and noise power. Solution: Assume $E[v[n]] = 0$, then

$$R_v[m] = \sigma_x^2 \delta[m], \qquad S_v(\omega) = \sigma_x^2$$

(d) $w[n] = h[n] * v[n]$ is a discrete-time narrowband noise source, where $h[n]$ is an ideal bandpass filter with center frequency $\pi/2$ radians/sample, and with bandwidth $\pi/10$ radians/sample. Compute the autocorrelation, power spectrum, and signal power. Solution:

$$S_w(\omega) = |H(\omega)|^2 S_v(\omega) = \begin{cases} \sigma_x^2 & 0.45\pi \le |\omega| \le 0.55\pi \\ 0 & \text{else} \end{cases}$$

$$R_w[m] = 0.1\sigma_x^2 \text{sinc}(0.05\pi m)\cos(0.5\pi m)$$
$$= 0.55\sigma_x^2 \text{sinc}(0.55\pi m) - 0.44\sigma_x^2 \text{sinc}(0.45\pi m)$$

Problem 7.4

Tones are presented embedded in a constant wideband noise source, of noise power N_0 Watts/m^2/Hz. The threshold intensity of the tone, I^*, is found to be a function of its center frequency f_c, as

$$10 \log_{10} I^*(f_c) = g(f_c)$$

What is the relationship between $g(f_c)$ and the equivalent rectangular bandwidth ERB(f_c)? Solution: Intensity of the effective noise masker is ERBN_0/π. Noise-masks tone threshold is about -6dB, so

$$10 \log_{10} I^* = 10 \log_{10}(N_0/\pi) + 10 \log_{10} \text{ERB} - 6$$

$$\text{ERB}(f_c) = \frac{4\pi}{N_0} 10^{g(f_c)/10}$$

Problem 7.5

Consider a T-network lumped element model of the cochlea, as in Fig. 3.2(b) of the Allen paper. For simplicity, assume that the mass of the cochlea is zero, i.e., $l_1 = l_2 = l_3 = \ldots = 0$. Assume also that the hair cell transducers $A_1, A_2, \ldots$ act like short circuits, i.e., they add no extra impedance to the circuit. Suppose that the circuit is excited by a wave $P_1(t) = A\cos(2\pi f_c t)$, where

$$2\pi f_c = \frac{1}{\sqrt{L_3 C_4}}$$

(a) Find the volume velocity at every branch in the circuit.

Solution: Superposition is the key to solving this problem without going insane. The flow through L_3 is equal to the flow through C_4, plus the flow through the rest of the circuit; let's write $U_{L_3} = U_{C_4} + U_{REM}$. The voltages and currents everywhere else in the circuit, including P_1, are composed of two terms: the term dependent on U_{C_4}, and the term dependent on U_{REM}. There is no easy way to solve for U_{REM} without numerical simulation, because it depends on a very large number of L-C pairs, all of different values, being driven by the (possibly very large) pressure $P_{C_4} = U_{C_4}/j\Omega_c C_4$. It is possible to write the anser to this problem in terms of U_{REM}, but that would not be very interesting, so let's calculate only the part depending on U_{C_4}.

At the input frequency $\Omega_c = 2\pi f_c$, the impedances of L_3 and C_4 are opposite, $j\Omega_c L_3 = -1/j\Omega_c C_4$. The pressure across C_3 is therefore $P_{C_3} = P_{C_4} + j\Omega_c L_3(U_{C_4} + U_{REM}) = j\Omega_c L_3 U_{REM}$. If we set $U_{REM} = 0$ (using superposition), then we get that $P_{C_3} = 0$. Therefore $U_{C_3} = 0$ (no flow through capacitor C_3), and $U_{L_2} = U_{L_3}$.

Under the assumption that $U_{REM} = 0$, the series connection of L_3 and C_4 has zero impedance. Therefore the total driving-point impedance of the circuit is

$$Z_1 = \frac{1}{j\Omega C_1} + j\Omega L_1 + \frac{j\Omega L_2}{1 - L_2C_2\Omega^2} = \frac{(1-\Omega^2 L_1C_1)(1-\Omega^2 L_2C_2) - \Omega^2 L_2C_1}{j\Omega C_1(1 - L_2C_2\Omega^2)}$$

The flow through L_1 is

$$U_{L_1} = \frac{P_1}{Z_1} = \frac{j\Omega C_1(1 - L_2C_2\Omega^2)A}{(1-\Omega^2 L_1C_1)(1-\Omega^2 L_2C_2) - \Omega^2 L_2C_1}$$

And the other branches in the circuit carry flows of

$$U_{C_2} == \frac{j\Omega C_1(-\Omega^2 L_2C_2)A}{(1-\Omega^2 L_1C_1)(1-\Omega^2 L_2C_2) - \Omega^2 L_2C_1}$$

and

$$U_{L_2} = U_{L_3} = U_{C_4} = \frac{j\Omega C_1 A}{(1-\Omega^2 L_1C_1)(1-\Omega^2 L_2C_2) - \Omega^2 L_2C_1}$$

(b) Find the pressure at every node in the circuit.

Solution: Pressure equals flow times impedance, so

$$P_{C_4} = \frac{U_{C_4}}{j\Omega C_4}$$

$$P_{C_3} = 0$$

$$P_{C_2} = \frac{U_{C_2}}{j\Omega C_2}$$

Problem 7.6

Assume that the cochlear map is as given in Eq. (3.12) of the Allen paper. Assume that a critical band corresponds to about 1mm on the cochlea. In this case, what is the critical bandwidth $\mathrm{ERB}(F_{cf})$ as a function of F_{cf}?

Solution: One ERB is ΔF when $\Delta X = 1$mm, therefore

$$\mathrm{ERB}(X) = -\frac{dF}{dX} = \frac{2.1\ln(10)}{L} 456 \times 10^{2.1(1-X/L)}$$

or, using the human cochlea values,

$$\mathrm{ERB}(X) = -\frac{dF}{dX} = \frac{2.1\ln(10)}{L} 165.4 \times 10^{2.1(1-X/L)}$$

In order to find ERB(F), we just need to substitute in $F(X)$, giving for the cat cochlea:

$$\mathrm{ERB}(F_c) = \frac{2.1\ln(10)}{L} 456 \times 10^{2.1(\frac{1}{2.1}\log_{10}(F_c/456+0.8))} = \frac{1}{4.3}(F_c + 365)$$

or for the human cochlea:

$$\mathrm{ERB}(F_c) = \frac{1}{7.2}(F_c + 146)$$